Stressing internal models with scenario weights

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Background

Collaborators

- **Alberto Bettini (Generali)**
- Vaishno Devi Makam (Bayes)
- **Pietro Millossovich (Bayes)**
- **Silvana Pesenti (Toronto)**
- Gabriel Seng (Bayes)
- Ruodu Wang (Waterloo)

Key papers for this talk

- Reverse sensitivity testing: What does it take to break the model?
- Scenario Weights for Importance Measurement (SWIM) – an R package for sensitivity analysis

What does it mean to explain a model?

*Und denkt man sich aus, **dass es eine Machine gäbe, deren Bauwerk es bewirke, zu denken, zu fühlen und Perzeptionen zu haben**, so wird man sich unter Beibehaltung der gleichen Maßstabsverhältnisse derart vergrößert vorstellen können, dass man sie wie in eine Mühle einzutreten vermöchte. Dies gesetzt, wird man in ihr, sobald man sie besucht, **nur Stücke finden, die einander stoßen, und nie etwas das eine Perzeption erklären möchte**.*

Leibniz (1714), *Monadologie*

How do we build trust?

Capital models [Cabantous & Tsanakas, 2021]

- Actuaries: sensitivity analyses, materiality of assumptions
- Underwriters: reflecting commercial views in parameterisation

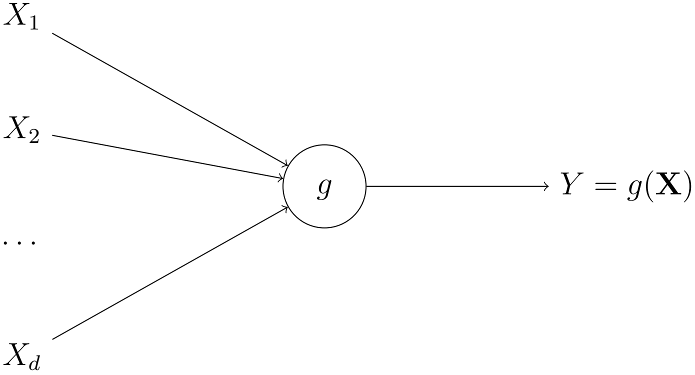
Predictive models

- Insurers: ability to tweak technical prices to reflect expert judgements
- Policyholders: right of redress

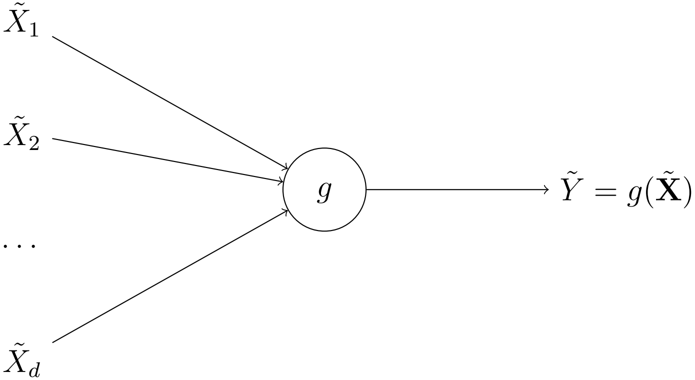
Key feature: **stakeholder agency**

- Model 'malleability'?

Internal models



Sensitivity to model changes



Evaluations of g are expensive

Scenario Weighting for Importance Measurement

Change of measure approach
[Meucci, 2008, Breuer et al., 2012, Cambou & Filipović, 2017, Pesenti et al., 2019, Pesenti et al., 2021a]

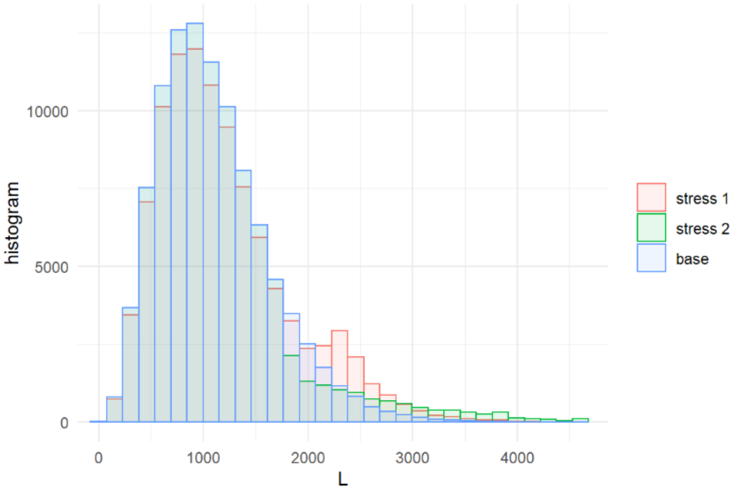
- Quantify response of the model to a specified **stress**
- A stress is understood as a measure change / vector of **scenario weights**

Scenario weights W obtained by **minimising KL-divergence**

$$\min_{W} \mathbb{E}\left[W \log W\right], \text{ subject to constraints on } \mathbf{X} \text{ under } \mathbb{P}^W$$

- **No need to re-run models**, ability to carry out large number of sensitivity tests
- **Non-linear aggregations** reflected by construction

The SWIM R package [Pesenti et al., 2021a]



Example

Simulated scenarios ($n = 10^5$) provided by a UK-based non-life insurer, from their stochastic economic capital model.

$$\mathbf{X} = (X_1, \ldots, X_{16}) : \text{ gross losses per line of business}$$

Y : net portfolio loss incl. non-insurance risks
$(SCR = \mathsf{VaR}_{0.995}[Y])$

$$Y = g(\mathbf{X}, \mathbf{V}), \text{ with } g \text{ a black box to us}$$

What stresses to apply?

Stressing gross volatilities

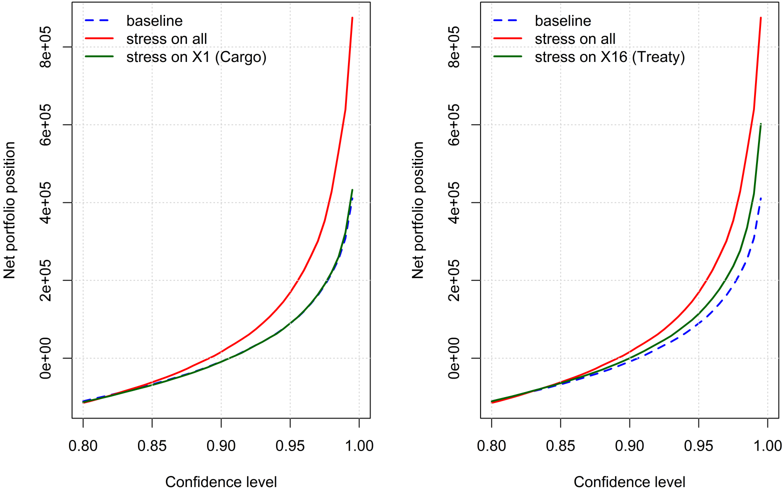
We begin by stressing all gross volatilities by 20%, while keeping means fixed (1 stress):

$$\mathbb{E}^{W}\left[X_i\right] = \mathbb{E}\left[X_i\right]$$
$$\sigma^{W}\left[X_i\right] = 1.2 \times \sigma\left[X_i\right], \; i = 1, \ldots, 16$$

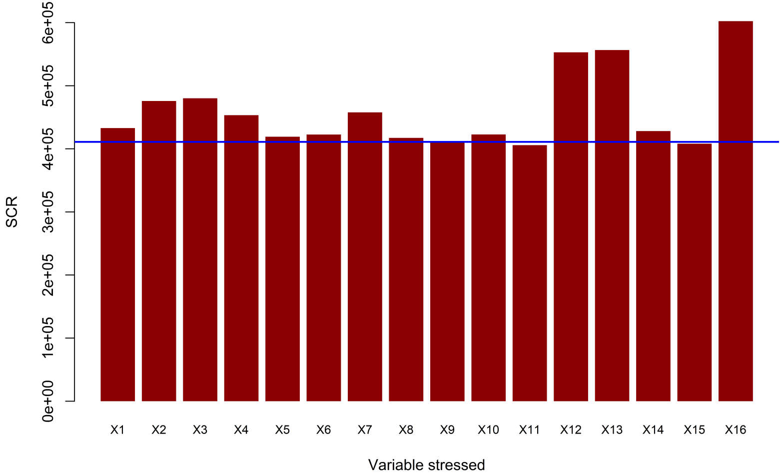
Subsequently we stress each gross volatility by 20%, one at a time (16 stresses):

$$\mathbb{E}^{W_i}\left[X_i\right] = \mathbb{E}\left[X_i\right]$$
$$\sigma^{W_i}\left[X_i\right] = 1.2 \times \sigma\left[X_i\right]$$

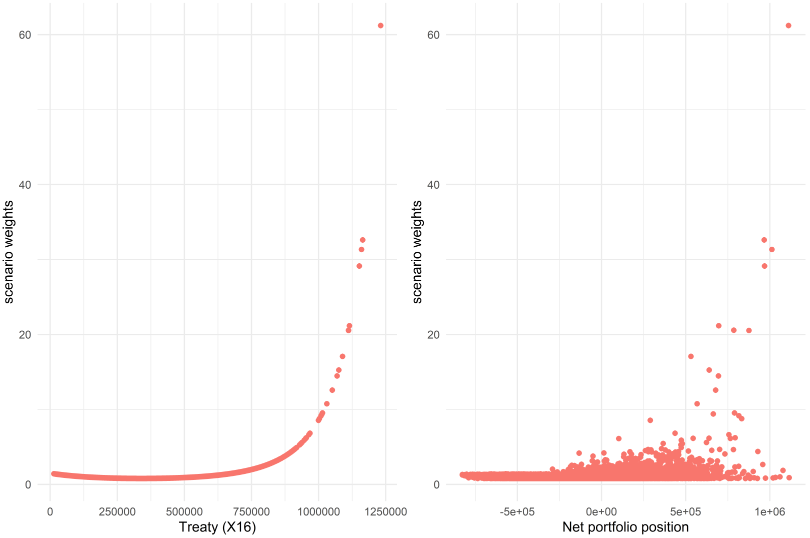
Comparing one-at-a-time stresses



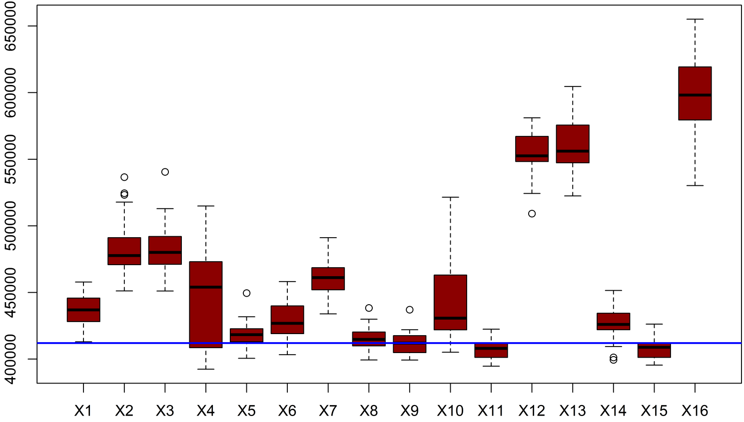
Impact on SCR (baseline in blue)



Scenario weights for stressing $\sigma[X_{16}]$



Simulation error



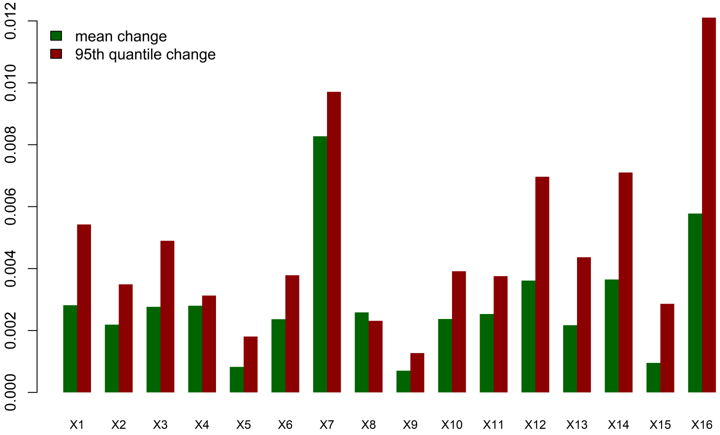
Reverse stress test

Let us now stress directly the portfolio loss, and monitor the impact on inputs

$$\mathsf{VaR}^{W}_{0.99}[Y] = 1.15 \times \mathsf{VaR}_{0.99}[Y]$$
$$\mathsf{ES}^{W}_{0.99}[Y] = 1.2 \times \mathsf{ES}_{0.99}[Y]$$

Record % changes in mean and 95th quantile of gross loss for each LoB

Reverse stress testing – impact on gross losses



What else?

With SWIM, each of the stresses above is just one line of R code

- Custom stresses on quantities such as Joint Exceedance Probabilities, one-sided moments, etc

Beyond (not yet in) SWIM

- Stressing multivariate distributions
- Cascading / indirect effects [Pesenti et al., 2021b]
- χ^2- rather than KL-divergence [Devi Makam et al., 2021]

Shiny app – coming soon

Stressing multivariate distributions

Work in progress with R. Wang and P. Millossovich (not yet in SWIM)

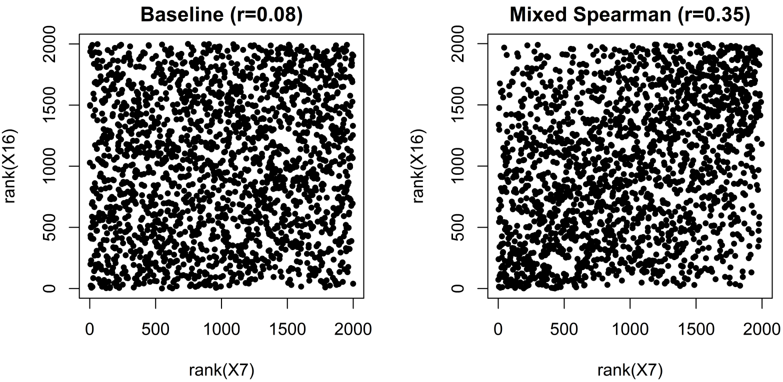
Proposition

Let $\mathbf{X}$ *be independent and* G *be a distribution on* $\mathbb{R}_+$ *with* $G(0) = 0$*. Define the weights*

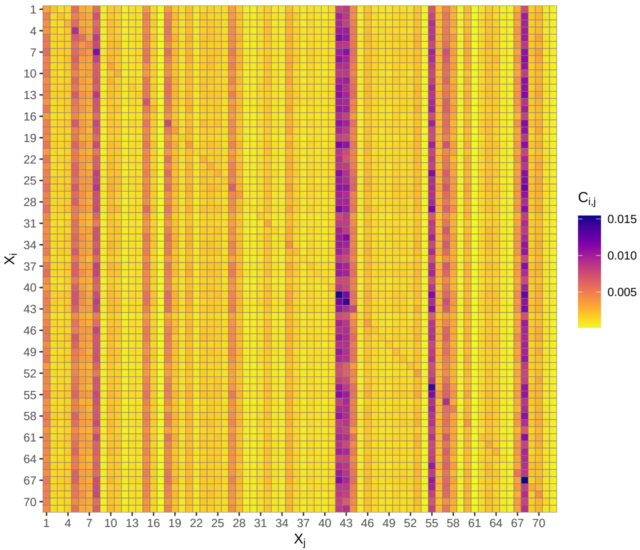
$$\eta(\mathbf{X}) = \int_0^\infty t^d \left(U_{X_1} \cdot \cdots \cdot U_{X_d}\right)^{t-1} dG(t).$$

Then, under $\mathbb{P}^{\eta(\mathbf{X})}$, $\mathbf{X}$ *has an Archimedean copula, with generator given by* $\phi(u) = \mathcal{L}(G)$.

A bivariate dependence stress



Cascading effects [Pesenti et al., 2021b]



Summary

Scenario weighting gives a **fast and practical** way to carry out sensitivity analyses of simulation models

- No extra model runs means that many more tests can be carried out $\rightsquigarrow$ **exploration**

SWIM implementation in R

- Inbuilt stresses and visualisations, flexible, free

Caveats

- Remember that we are **limited to a given table of simulations**, cannot reflect events that are not present
- Stressing with SWIM is **not** the same as changing the **parameters** of a distribution (though can be close)

THANK YOU FOR YOUR ATTENTION!

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